| Tanta University | Department: Engineering Physics and Mathematics<br>Total Marks: 85 Marks | Faculty of Engineering |
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Course Title: Mathematics(3B)          Course Code: PME2211          Year: 2nd( Computer &Control Dep.)
Date: June 8th, 2014 (Second term)    Allowed time: 3 Hrs              No. of Pages: (3)

**Remarks:  Answer  All of The Following Questions**

## Question Number (1)                    (25 Marks)

( a) Show that the set     $A = \int \left(\frac{1}{\sqrt{1+5x}}\right)/x$   is convex

(b ) Consider the fuzzy sets F and G defined in interval [0,10] by the memberships

$\mu_F(x) = 2^{-x}$ and  $\mu_G(x) = \frac{1}{1+10(x-2)^2}$ .Determine the mathematical formulas and graphs

of  memberships functions of        (i ) $\mu_{\bar{F}}$ and $\mu_{\bar{G}}$        (ii ) $\mu_{F \cup G}$  and $\mu_{F \cap G}$

(c )   A product with memberships represents, degree of high expensive   $\mu_A(x)$ , degree of medium expensive   $\mu_B(x)$ and degree of cheap  expensive  $\mu_c(x)$ .Us defuzzification methods to find suitable price, if   its medium degree is 0.5 and high degree 0.8 where
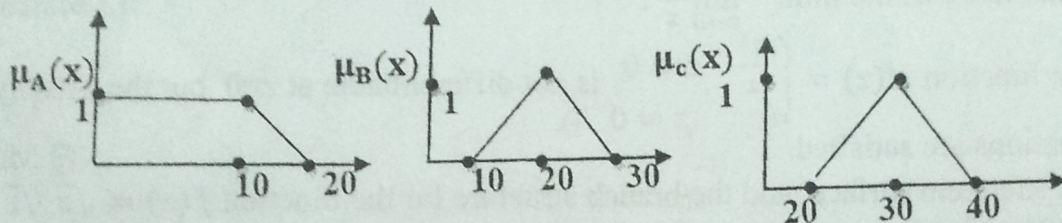


( d)   Consider Washing machine with  two input and one output . The input :

[1] The dirtiness of the Load which measured by the opacity of the  washing water use an optical sensor system {Almost clean(AC), Dirty(D), Soiled(S), Filthy(F)}with fuzzy dirtiness membership   $\mu_A(x)$

[2]  The weight of the Laundry load as measured by  a pressure sensor system {Very light(VL), Light(L), Heavy(H), Very heavy(VH)} with fuzzy weight membership $\mu_B(x)$
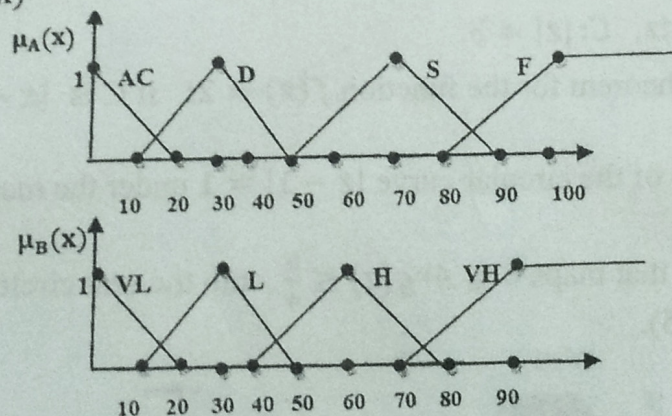


The output is the amount of detergent dispensed {Very little(VL), Liittle(L), Much(M), Very much(VM), Maximum(Max)}  $\mu_C(x)$
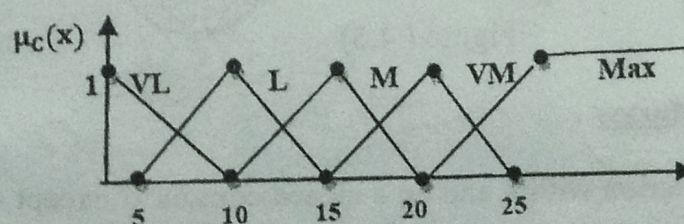


Find the fuzzy detergent dispensed value if laundary has dirtiness values 13 and weight 72

## Question Number (2)          (15 Marks)

If the method of Frobenius is used to solve the following linear homogenous $2^{nd}$ order ordinary differential equation (L.H.O.D.E.)

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + (x^2 + 1)y = 0$$

1. Show that the point x=0 is the unique regular singular point.                    (3 Marks)
2. Deduce the indicial equation and find the values of $\lambda$.                    (4 Marks)
3. Deduce the recurrence relation for coefficient of the series solution.            (4 Marks)
4. Deduce the general series solution $y(x)$.                    (4 Marks)

## Question Number (3)          (15 Marks)

1. Sketch the domain of the function $f(z) = \frac{1}{3(z^2 + \bar{z}^2) - 10z\bar{z} + 25}$.                    (3 Marks)

2. Prove that $f(z) = \frac{z}{z^4 + 1}$ is continuous at all points inside and on the unit circle $|z| = 1$ except at some points and determine these points.                    (3 Marks)

3. Discuss the existence of the limit $\lim_{z \to 0} \frac{z^2}{\bar{z}}$.                    (3 Marks)

4. Show that the function $f(z) = \begin{cases} \frac{\bar{z}^2}{z} & z \neq 0 \\ 0 & z = 0 \end{cases}$ is not differentiable at z=0 but the Cauchy-Riemann equations are satisfied.                    (3 Marks)

5. Construct the Riemann surface and the branch structure for the function $f(z) = \sqrt{z} \sqrt[3]{z}$ if the angle is embedded in $[-\pi, \pi]$.                    (3 Marks)

## Question Number (4)          (15 Marks)

1. Let f(z) be an analytic in a simple connected domain and let C be a simple closed contour lying entirely within the domain, if $z_0$ is any point interior in C then prove that
   $2\pi i f^{(n)}(z_0) = n! \oint_C \frac{f(z)}{(z - z_0)^{n+1}} dz, \quad n = 0,1,2,3, \dots$.                    (3 Marks)

2. Evaluate $\oint_C \frac{Ln(z)}{(z-3)^3 (z+4)^2} dz, \quad C: |z| = 3$.                    (3 Marks)

3. Verify Cauchy's integral theorem for the function $f(z) = 2i$ if C is $|z - 3i| + |z + 3i| = 20$.                    (3 Marks)

4. Find and sketch the image of the circular curve $|z - 1| = 1$ under the mapping $w = \frac{1}{z}$.                    (3 Marks)

5. Find the bilinear mapping that maps $0 \leq Arg\{z\} \leq \frac{\pi}{4}$ onto the unit circle $|w| \leq 1$ as shown in the following figure (4.5).                    (3 Marks)
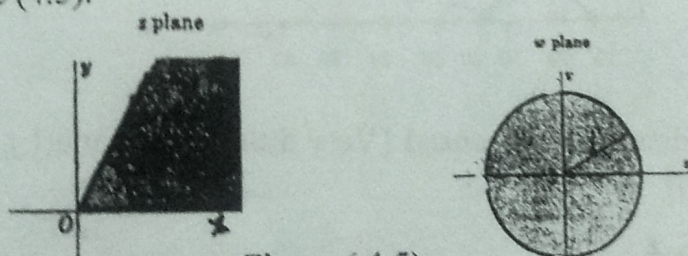


Figure ( 4.5)

## Question Number (5)          (15 Marks)

1. Let f(z) be an analytic function within and on a closed contour C except at a finite number of singular points $z_1, z_2, \dots, z_n$ interior to C. Then prove $\oint f(z) dz = 2\pi i \sum_{i=1}^{n} \operatorname*{Res}_{z=z_i}\{f(z)\}$ where the integral is taken counter clockwise direction around C.                    (3 Marks)

2. Find the Laurent series for $f(z) = \frac{z - e^{2z}}{z-1}$ about z=1 then name the singularity and give the region of the convergence. (3 Marks)

3. Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series valid for $0 < |z + 1| < 2$ and find its residue. (3 Marks)

4. Using the residue theorem to evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$. (3 Marks)

5. Using the residue theorem to evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2(x^2+2x+2)} dx$. (3 Marks)

**With Best Wishes**

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**Course Examination Committee and Course Coordinators**

Dr. Mohamed Shokry and   Dr. Mohamed Elborhamy    and the committee